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Stability Analysis of a Mathematical Model on the Revitalization Efforts of Local Languages in East Kalimantan

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ABSTRACT

Local languages are an essential part of cultural identity, but are increasingly endangered due to modernization and globalization. This study aims to develop a mathematical model based on a dynamic system to describe the revitalization of local languages in East Kalimantan. The model divides the population into four compartments: individuals who cannot speak the local language (S), participants in local language education (P), language promoters (R), and technology users in local language learning (T). The model assumes a closed population with key parameters, including the education transmission rate (β), transition to promotion (γ), and the technology utilization rate (δ). The analysis involves determining the equilibrium points, calculating the basic reproduction number (R_0) using the Next Generation Matrix method, and assessing the local stability of the system. The results show that $R_0 > 1$ indicates the potential sustainability of local languages, while $R_0 < 1$ suggests a tendency toward extinction. Simulations reveal that increasing language promotion and technology utilization can significantly raise R_0, making their combination an effective strategy to maintain the vitality of local languages.

Keywords: *Local Language Revitalization, Mathematical Modeling, Dynamic Systems, Reproduction Number* (R_0), *Stability Analysis.*

1. INTRODUCTION

Language is not merely a tool for communication but also a marker of cultural identity and collective memory within a community [2], [3]. The loss of a local language signifies not only linguistic decline but also the erosion of cultural heritage and traditional knowledge systems [4]. According to national and international reports, more than forty percent of the world's languages are currently endangered, primarily due to reduced numbers of native speakers and weakened intergenerational transmission [1], [5].

Indonesia, as a linguistically diverse nation, faces serious challenges in maintaining its local languages. In East Kalimantan, several regional languages such as Paser, Tunjung, Dayak Benuaq, Tidung, and Kutai are experiencing declining vitality [5], [17]. To address this issue, the government has implemented local language revitalization programs through formal education, community-based promotion, and the utilization of digital technology [6]–[8], [17]. However, systematic evaluation of the long-term effectiveness of these programs remains limited.

Interdisciplinary approaches are therefore required to analyze the dynamics of language revitalization. Mathematical modeling, particularly compartmental models widely used in epidemiology, provides a robust framework for understanding transmission processes within populations [9], [12]. In this context, language acquisition and use can be interpreted as a form of cultural transmission analogous to the spread of ideas or behaviors [18].

This study proposes a mathematical model to analyze local language revitalization efforts in East Kalimantan. The objectives are to construct a dynamic system representing language transmission, derive a threshold parameter for sustainability, analyze equilibrium stability, and provide insights relevant to language policy and revitalization strategies.

2. LITERATURE REVIEW

This literature review synthesizes interdisciplinary studies related to local language endangerment and revitalization. It integrates perspectives from sociolinguistics, mathematical modeling, and digital technology to highlight key factors influencing language sustainability. By reviewing prior empirical findings and theoretical frameworks, this section establishes a scientific basis for analyzing language transmission dynamics and evaluating revitalization strategies within contemporary social contexts.

2.1 Local Language Endangerment and Revitalization

Previous studies indicate that the decline of local languages is closely related to limited use in formal domains, reduced family transmission, and dominance of national or global languages [4], [5]. Revitalization efforts through education and community engagement have been shown to improve language awareness and usage among younger generations [6], [7]. Policy-oriented studies further emphasize the importance of institutional support and curriculum integration in sustaining local languages [8], [17].

2.2 Mathematical Modeling of Cultural and Linguistic Transmission

Mathematical models have long been applied to population dynamics and epidemiology to study transmission mechanisms [9], [12], [13]. Recent research extends these approaches to cultural and linguistic contexts, demonstrating that threshold parameters such as the basic reproduction number play a crucial role in determining sustainability [14], [15], [18]. Such models provide a quantitative basis for evaluating intervention strategies.

2.3 Technology and Digital Media in Language Revitalization

The use of digital technology, including online dictionaries, learning applications, and social media platforms, has emerged as an effective strategy for language revitalization [7], [18]. Technology expands access to learning resources and enhances language visibility, particularly among younger speakers who are more engaged with digital media.

3. METHODS

The mathematical model for revitalizing local languages in East Kalimantan constructs a closed population consisting of four compartments. The population of **individuals (S)** represents native residents who can no longer speak their local language. The population of **learners (P)** represents individuals who gain knowledge of local languages through revitalization programs conducted via education and training. The **promotion group (R)** consists of individuals affected by the revitalization process who, after participating in education and training programs, engage in promoting local languages. The **technology users (T)** represent individuals who adopt technology as a means of promoting and teaching local languages.

The total population is given by:

$$\boldsymbol{N = S + P + R + T}.$$

The mathematical model of local language revitalization in East Kalimantan is developed under the following assumptions:

1. The population is **closed** (no migration in or out).
2. Populations $\boldsymbol{P}$, $\boldsymbol{R}$, and $\boldsymbol{T}$ are influenced by local language revitalization activities.
3. Individuals who are unable to speak the local language can acquire it through education and training programs, with a transmission rate denoted by $\boldsymbol{\beta}$.
4. The natural birth and death rates of individuals are assumed to be equal, represented by $\boldsymbol{\mu}$.
5. Only individuals who have participated in education and training can transition to the promotion group ($\boldsymbol{R}$) at a rate $\boldsymbol{\gamma}$.
6. Only individuals from the promotion group can transition to the technology group ($\boldsymbol{T}$) at a rate $\boldsymbol{\delta}$.

Based on the assumptions above, the mathematical model for revitalizing local languages in East Kalimantan can be illustrated through the compartment diagram shown in Figure 1.

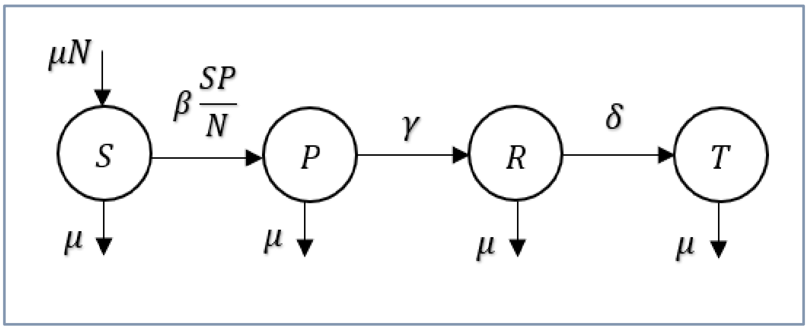

Figure 1. Compartment diagram of the mathematical model for the revitalization efforts of local languages in east kalimantan

Based on the compartment diagram above, a differential equation system can be written from the mathematical model for regional language revitalization efforts in East Kalimantan with the following scaling:

$$\begin{aligned}\frac{dS}{dt} &= \mu + \beta SP - \mu S \\ \frac{dP}{dt} &= \beta SP - \gamma P - \mu P \\ \frac{dR}{dt} &= \gamma P - \delta R - \mu R \\ \frac{dT}{dt} &= \delta R - \mu T\end{aligned} \qquad (1.1)$$

The description of the parameters used in the mathematical model for revitalizing local languages in East Kalimantan is presented in Table 1.

Table 1. Presents the description of parameters used in the mathematical model for local language revitalization in East Kalimantan.

Parameter	Description	Value	Source
μ	The average natural birth and death rate of individuals	$\frac{1}{70 * 365}$	[11]
β	Transmission rate from individuals unable to speak the local language to those participating in local language revitalization education programs	$0.005\ (E^0)$ $0.5\ (E^*)$	Assumptions

γ	Transition rate of individuals moving from education and training programs to the group promoting local language use	0.10 (E^0) 0.06 (E^*)	Assumptions
δ	Transition rate of individuals moving from the promotion group to those utilizing technology for local language learning and promotion	0.0333 (E^0) 0.014 (E^*)	Assumptions

4. RESULTS AND DISCUSSION

The model analysis identifies two equilibrium points: a local-language-free equilibrium and a local-language-existence equilibrium. Stability analysis shows that the language-free equilibrium is locally asymptotically stable when the basic reproduction number is less than one, indicating eventual language extinction [12], [13]. Conversely, when the reproduction number exceeds one, the existence equilibrium becomes stable, signifying sustainable language revitalization.

Numerical simulations support the analytical results and demonstrate that increasing participation in education programs, strengthening promotion activities, and enhancing technology utilization significantly improve language sustainability. These findings are consistent with empirical studies emphasizing the combined roles of education, community engagement, and digital tools in revitalizing endangered languages [6],[7],[8],[18].

Model Analysis

In this section, we analyze the previously developed mathematical model for the revitalization efforts of local languages in East Kalimantan by determining the equilibrium points of the system, calculating the basic reproduction number (R_0), examining the stability of each equilibrium point, and performing numerical simulations of the model.

The equilibrium points are obtained by considering the system in a steady-state condition, that is, when $\frac{dS}{dt} = 0$, $\frac{dP}{dt} = 0$, $\frac{dR}{dt} = 0$, and $\frac{dT}{dt} = 0$. Based on this, two equilibrium points are identified: the **local language-free equilibrium point (E^0)** and the **local language existence equilibrium point (E^*).**

4.1 Local Language-Free Equilibrium Point (E^0)

The local language-free equilibrium point is given by:

$$\boldsymbol{E^0 = (S^0, P^0, R^0, T^0) = (1, 0, 0, 0).}$$

This equilibrium represents a state in which no individuals are capable of speaking the local language through revitalization efforts, indicated by $P^0 = 0$, $R^0 = 0$, and $T^0 = 0$. Meanwhile, the population of individuals who are unable to speak the local language persists in the system, represented by $S^0 = 1 > 0$.

1. Basic Reproduction Number (R_0)

From the local language-free equilibrium point (E^0), the basic reproduction number (R_0) of the model can be determined. This parameter is used to measure and analyze the dynamics of local language dissemination in East Kalimantan within the context of revitalization efforts aimed at preserving linguistic sustainability.

In this study, R_0 is interpreted as an indicator of the ability of local languages in East Kalimantan to spread and persist within the population through government-initiated revitalization programs. The basic reproduction number (R_0) is obtained using the **Next Generation Matrix (NGM)** approach by considering the populations exposed to local language influence—namely, P, R, and T.

Accordingly, the **transition matrix** (Σ) and the **transmission matrix** (**T**) are constructed as follows:

$$\mathbf{\Sigma} = \begin{bmatrix} -\gamma-\mu & 0 & 0 \\ \gamma & -\delta-\mu & 0 \\ 0 & \delta & -\mu \end{bmatrix} \text{dan } \mathbf{T} = \begin{bmatrix} \beta & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The construction of the Next Generation Matrix (NGM) is derived using the following formula:

$$\boldsymbol{NGM} = -\mathbf{T}\mathbf{\Sigma}^{\mathbf{1}} = \begin{bmatrix} \dfrac{\beta}{\gamma+\mu} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Using the method proposed by **van den Driessche [12]** and **Diekmann [13, 15]**, it is known that the basic reproduction number (R_0) is defined as the **spectral radius** of the Next Generation Matrix (NGM), that is, the maximum absolute value of its eigenvalues [14]. Thus, we obtain:

$$R_0 = \frac{\beta}{\gamma+\mu}.$$

Therefore, the contact rate between individuals who are unable to speak the local language and those participating in revitalization education programs, the transition rate of individuals moving from training participants to language promotion groups, and the natural mortality rate of individuals significantly influence the level of dissemination and development of local languages through the revitalization programs implemented in East Kalimantan.

In general, if $R_0 > 1$, the local languages in East Kalimantan have the potential to spread and continue to develop within the population. Conversely, if $R_0 < 1$, the local languages will gradually decline over time, which may ultimately lead to their extinction.

2. Analysis of the Local-Language-Free Equilibrium Point ($\boldsymbol{E^0}$)

The analysis of the local-language-free equilibrium point (E^0) is carried out by evaluating the equilibrium point (E^0) in the Jacobian matrix (J) as follows.

Theorem 1. *Stability of the Local-Language-Free Equilibrium point (E^0) is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.*

Proof. From the Jacobian matrix (J) for the system is given by:

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial S} & \frac{\partial f_1}{\partial P} & \frac{\partial f_1}{\partial R} & \frac{\partial f_1}{\partial T} \\ \frac{\partial f_2}{\partial S} & \frac{\partial f_2}{\partial P} & \frac{\partial f_2}{\partial R} & \frac{\partial f_2}{\partial T} \\ \frac{\partial f_3}{\partial S} & \frac{\partial f_3}{\partial P} & \frac{\partial f_3}{\partial R} & \frac{\partial f_3}{\partial T} \\ \frac{\partial f_4}{\partial S} & \frac{\partial f_4}{\partial P} & \frac{\partial f_4}{\partial R} & \frac{\partial f_4}{\partial T} \end{bmatrix}$$

where f_1, f_2, f_3, and f_4 denote the right-hand sides of the differential equations corresponding to the compartments S, P, R,and T, respectively.

By substituting the equilibrium point $E^0 = (S^0, P^0, R^0, T^0) = (1,0,0,0)$into the Jacobian matrix, we obtain:

$$J(E^0) = \begin{bmatrix} -\mu & -\beta & 0 & 0 \\ 0 & \beta-(\gamma+\mu) & 0 & 0 \\ 0 & \gamma & -(\delta+\mu) & 0 \\ 0 & 0 & \delta & -\mu \end{bmatrix}$$

The local stability of the equilibrium point E^0 can be determined from the eigenvalues of the Jacobian matrix $J(E^0)$. If all eigenvalues have negative real parts, the equilibrium is locally asymptotically stable; otherwise, it is unstable.

The characteristic equation of the Jacobian matrix is:

$$(\lambda+\mu)(\lambda-[\beta-(\gamma+\mu)])(\lambda+\delta+\mu)(\lambda+\mu)=0$$

Hence, the eigenvalues are obtained as:

$$\begin{aligned}\lambda_1 &= -\mu,\\ \lambda_2 &= \beta-(\gamma+\mu)\\ \lambda_3 &= -(\delta+\mu)\\ \lambda_4 &= -\mu\end{aligned}$$

It can be observed that all eigenvalues are negative except λ_2, which depends on the relation between β and $(\gamma+\mu)$:

a. If $\beta<(\gamma+\mu)$, then $\lambda_2<0$, and all eigenvalues are negative, implying that the equilibrium point E^0 is **locally asymptotically stable**. This condition corresponds to $R_0<1$, indicating the potential extinction of the local language.
b. Conversely, if $\beta>(\gamma+\mu)$, then $\lambda_2>0$, and the equilibrium becomes **unstable**, corresponding to $R_0>1$, indicating that the local language can persist and spread within the population.

4.2 Existence Equilibrium Point of the Local Language (E^*)

The existence equilibrium point of the local language (E^*)is expressed as:

$$\begin{aligned}E^* &= (S^*,P^*,R^*,T^*)\\ &= \left(\frac{\gamma+\mu}{\beta},\frac{(\beta-\gamma-\mu)\gamma\mu}{\beta(\gamma+\mu)},\frac{(\beta-\gamma-\mu)\mu}{\beta(\delta\gamma+\delta\mu+\gamma\mu+\mu^2)},\frac{(\beta-\gamma-\mu)\delta\gamma}{\beta(\delta\gamma+\delta\mu+\gamma\mu+\mu^2)}\right).\end{aligned}$$

This equilibrium point represents the condition in which all populations involved in the revitalization process of the local language continue to exist within the system. Specifically, it indicates that the populations of individuals who participate in language education and training (P^*), those involved in the promotion of the local language (R^*), and those who use technology to promote and teach the language (T^*) remain positive, i.e., $P^*>0$, $R^*>0$, and $T^*>0$. Hence, the existence of the equilibrium point $E^*$is guaranteed when the basic reproduction number satisfies $R_0>1$.

In other words, the condition $R_0>1$ implies that the local language in East Kalimantan has the potential to **spread, sustain, and develop** within the population through the implemented revitalization programs. Conversely, if $R_0<1$, the revitalization efforts will not be sufficient to maintain the use of the local language, leading to its gradual decline.

1. Analysis of the Existence Equilibrium Point (E^*)

The analysis of the existence equilibrium point (E^*) is conducted by evaluating the equilibrium point E^*on the Jacobian matrix J defined previously.

Theorem 2. *Stability of the Local-Language Existence Equilibrium point* (E^*) exists and is **locally asymptotically stable** if $R_0>1$.

Proof. Substituting the existence equilibrium point values (E^*) into J, we obtain the following characteristic equation:

$$P^*(\lambda)=(\lambda+\mu)(\lambda+\delta+\mu)(a_2\lambda_2-a_1\lambda_1+a_0)$$

where the coefficients are defined as:

$$a_2 = -(\gamma + \mu)$$
$$a_1 = -\beta\mu$$
$$a_0 = (\gamma - \beta + \mu)(\gamma + \mu)$$

From this characteristic polynomial, the eigenvalues of the Jacobian matrix are given by:

$$\lambda_1 = -\mu$$
$$\lambda_2 = -(\delta + \mu)$$

According to **Descartes' Rule of Signs**, the quadratic polynomial $(a_2\lambda^2 - a_1\lambda + a_0)$ will have negative real roots, i.e., $\lambda_{3,4} < 0$, if all coefficients a_2, a_1, and a_0 maintain the same sign. It is known that $a_2 < 0$and $a_1 < 0$. For $a_0 < 0$, the condition is satisfied when the basic reproduction number meets $R_0 = \frac{\beta}{\gamma+\mu} > 1$.

Therefore, the **existence equilibrium point** (E^*) is **locally asymptotically stable** when $R_0 > 1$. This condition implies that the local language in East Kalimantan will continue to **exist and thrive** within the population through sustained revitalization efforts.

Model Simulation

Numerical simulations are conducted to illustrate the dynamic changes in each population compartment involved in the revitalization efforts of local languages in East Kalimantan, with the time unit measured in days. The parameters used in the simulation are based on those listed in Table 1.

4.3 Simulation of the Local-Language-Free Equilibrium (E^0)

The simulation for the local-language-free equilibrium point $(E^0)$is performed using the initial values $[S^0 = 0.94, P^0 = 0.005, R^0 = 0.05, T^0 = 0.005]$ with $R_0 < 1$.

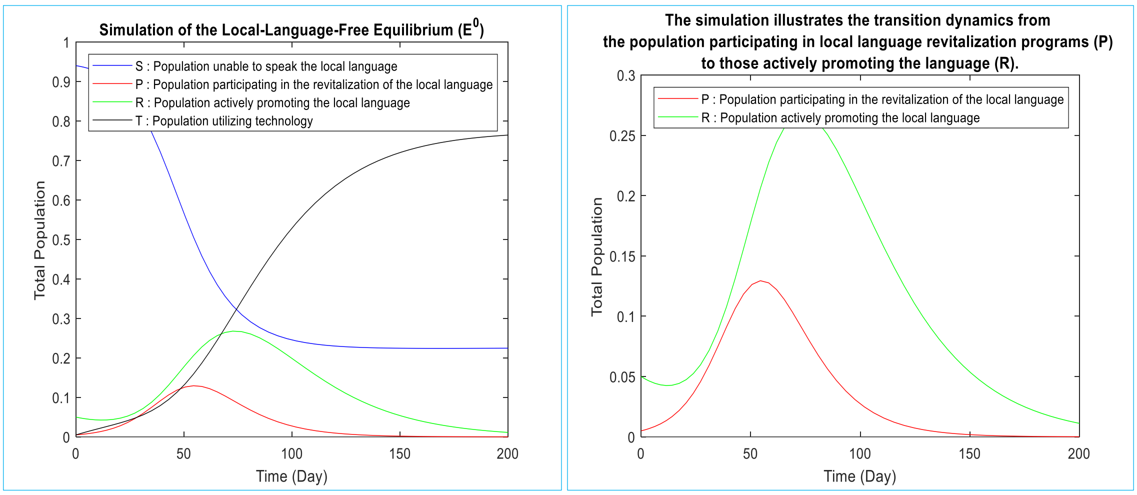

Figure 2. Mathematical Model Simulation of the Local-Language-Free Equilibrium (E^0)

The simulation results for $R_0 < 1$show that the populations in the compartments $P(t)$, $R(t)$, and $T(t)$— representing individuals involved in education, promotion, and technology-based learning of the local language — gradually decline to zero over time. Conversely, the proportion of individuals unable to speak the local language, $S(t)$, approaches one, indicating dominance of the non–local-language-speaking group.

This behavior confirms the analytical result that when $R_0 < 1$, the local-language-free equilibrium (E^0) is locally asymptotically stable. In this condition, revitalization efforts fail to sustain language transmission, leading to the gradual extinction of the local language within the population.

4.4 Simulation of the Local-Language Existence Equilibrium (E^*)

The simulation for the local-language existence equilibrium point (E^*)is conducted using the initial values $[S^* = 0.5, P^* = 0.3, R^* = 0.1, T^* = 0.1]$, while increasing the transmission rate βsuch that $R_0 > 1$.

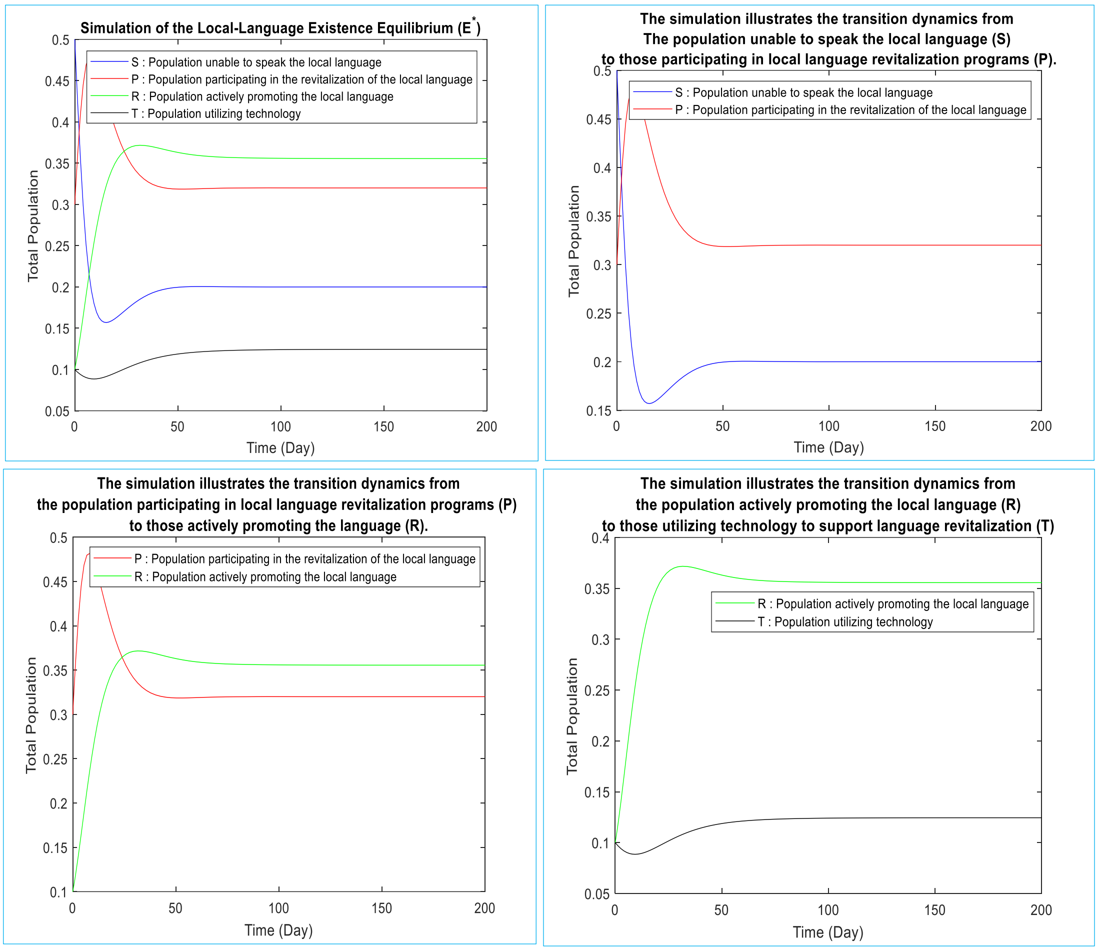

Figure 3. Mathematical Model Simulation of the Local-Language Existence Equilibrium (E^*)

The simulation results show that the populations in $P(t)$, $R(t)$, and $T(t)$— representing individuals involved in education, promotion, and technology-based language learning — converge to positive steady-state values over time. Meanwhile, $S(t)$ representing individuals unable to speak the local language, gradually decreases to a lower equilibrium level.

The simulation demonstrates a **stable equilibrium point** E^*, indicating that the local language persists with a constant proportion of active users. This result highlights that the combination of increasing the **language education rate (β)**, **promotion rate (γ)**, and **technology utilization rate (δ)** plays a crucial role in sustaining the existence of local languages in East Kalimantan.

CONCLUSION

This study formulates a compartmental dynamic system model as an interdisciplinary analytical framework to examine the revitalization of local languages in East Kalimantan. By representing population groups through education participation, promotion activities, and technology-based language use, the model captures the interaction between social, cultural, and technological processes. The analytical results reveal that the long-term behavior of the system is governed by the basic reproduction number R_0. A value of $R_0 < 1$ leads to a stable local-language-free equilibrium, indicating language decline, whereas $R_0 > 1$ ensures a stable existence equilibrium, signifying sustained language vitality. Numerical simulations further validate that increases in education, promotion, and technology utilization rates enhance system stability and support language persistence.

From an interdisciplinary perspective, the proposed model demonstrates how mathematical dynamical systems can be effectively employed to study sociolinguistic revitalization processes. The results highlight that the integration of formal education, social promotion mechanisms, and digital technology forms a coherent and sustainable structure for maintaining local languages. This framework provides a quantitative foundation for future empirical studies and cross-disciplinary collaboration, enabling the development of data-driven strategies for language preservation in the context of cultural transformation and globalization.

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